ZU-TH 25/94
October 1995

Unitary Background Gauges and Hamiltonian approach to Yang–Mills Theories [1]

A. Dubin[a] and D. Wyler[b]

[a] Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia
e-mail address dubin@vxitep.itep.su

[b] Universität Zürich, 8057 Zürich, Switzerland
e-mail address wyler@physik.unizh.ch

Abstract

A variety of unitary gauges for perturbation theory in a background field is considered in order to find those most suitable for a Hamiltonian treatment of the system. We select two convenient gauges and derive the propagators $D_{\mu\nu}$ for gluonic quantum fluctuations immersed in background configurations. The first one is a unitary generalization of the usual Coulomb gauge in QED which preserves the decoupling of two propagating polarizations from the instantaneous one. The second possibility is the axial light cone gauge which remains ghost free also in the presence of a background. Applications of the formalism to the spectrum and dynamics of QCD at the confinement scale, such as hybrid states, are briefly discussed.

[1]partially supported by Schweizerischer Nationalfonds.

1 Introduction

The identification of relevant degrees of freedom [1, 2, 3] in the non-perturbative region of QCD is the crucial step towards a consistent theory of hadrons. One of the ways [1, 2] to disentangle this problem in an economical way is to separate in Euclidean space first non-trivial 'vacuum' configurations B_μ, e.g. to split the total gluonic field A_μ into a background and a quantum fluctuation

$$A_\mu = B_\mu + a_\mu \tag{1.1}$$

where the field B_μ is primarily responsible for the long range gluonic correlation functions. The qualitative picture for B_μ–fields is the ensemble of lumps described by their collective coordinates. For the self consistency of the separation (1.1), these collective coordinates should not be distorted after inclusion of quantum fluctuations a_μ.

Recently, this idea has been emphasized [4, 5], with the suggestion to consider perturbation theory in a confining background which leads to the area law asymptotics of the averaged Wilson loop. The (path) integral over the background fields can be reformulated [6] as a summation over irreducible correlators of vacuum field strength tensors by means of the cluster expansion. In this way, the existence of flux tubes between quarks is related to decaying correlators with particular Lorentz structures [6]. Although the procedure appears reasonable, and is supported by the results from the simpler $2+1$ compact $U(1)$ theory [2] where the monopole background fields generate at large distances the 'frozen' string, no consistent theory exists for such background fields in QCD. At present, one must *postulate* that the dynamics of the vacuum configurations leads to a finite string tension, which can be expressed as the infinite sum over contributions from the vacuum correlators of all orders.

Since the confining background cures all infrared singularities [4], it enables one to investigate the dynamics of the a_μ–perturbations at large distances as well. The important point is that the a_μ–fluctuations immersed into such vacuum fields describe the effective long distance excitations of the string [4, 5] frozen at the level of the vacuum contribution. They should be considered, besides quarks, as the relevant degrees of freedom at the scale of confinement. For instance, the interpretation of the one a_μ-gluon exchange amplitude (averaged over the B_μ-fields) of Fig.1a requires introducing the first excitation of the string as in Fig. 1b. In general, the QCD string between quarks will be divided by valence gluons into a corresponding number of elementary 'frozen' pieces.

Within this picture, the basic question is how to describe the gluonic excitations ('constituent gluons') a_μ interacting via the background in a nonperturbative way. Because of gauge invariance, a_μ in eq.(1.1) contains (unphysical) pure gauge components. Also the presence of non-decoupled propagating ghosts inherent in the standard formulation [7, 1, 2] of perturbation theory in a background field with the usual gauge condition

$$D_\mu(B)a^\mu = 0 \tag{1.2}$$

will obscure the analysis of bound states. It is the main goal of this paper to explore these problems and to select unitary background gauges where ghosts are either absent or nonpropagating. This will allow us to describe bound states in terms of physical polarizations of a_μ only.

To apply the propagators $D_{\mu\nu}(B) =< a_\mu a_\nu >$ to a given problem, we suggest to use the multichannel Hamiltonian approach [8, 9] recently generalized [10] to this kind of fluctuating

strings. It provides a scheme to derive from the propagator both the diagonal elements of the Hamiltonian for states with a fixed number of propagating constituent gluons as well as the nondiagonal terms which mix different Fock states. In this way, we can investigate glueballs or hybrids. We stress that $D_{\mu\nu}(B)$ provides us with both spin-dependent and spin-independent nonperturbative interactions between a_μ-gluons and other constituents at the scale of confinement. We note that the diagonal approximation for such states based on the standard nonunitary background gauge (1.2) (with the propagating ghosts) has recently been suggested in ref. [5].

The plan of the paper is as follows. In the next section, we discuss further the special role of the unitary background gauges for the analysis of QCD bound states. Then, in section 3, we review the derivations of $D_{\mu\nu}$ in the abelian case within the path integral approach.

In section 4, the standard analysis of perturbation theory in background fields is extended to prove the well known background gauge invariance [7, 1, 2] for the case of the special unitary gauges to be described in sections 5 and 6. There, the explicit form of the propagators $D_{\mu\nu}(B)$ will be derived. Finally section 7 contains a brief sketch of the applications and the conclusions.

2 The special role of unitary background gauges

Before constructing explicitly the propagators in the unitary background gauges, we discuss in more detail why they are important for the analysis of gluonic excitations in bound states.

As stressed above, the key point is that if dynamical gluons appear as constituents of a bound state, such as in glueballs or in hybrid states, non-physical polarizations of $D_{\mu\nu}$ (which occur in a nonunitary gauge) obscure both the interpretation of the quantum numbers of a hadron *and* the dynamical scheme of interactions between the valence constituents. Due to gauge invariance, the contributions from 'pure gauge' polarizations should drop out (together with ghost contributions) in the final result. But in order to invoke physical intuition, it is important to have the transparent and economical formulation which is provided by unitary gauges where ghosts are either absent or nonpropagating. Then all independent propagating polarizations in $D_{\mu\nu}$ can be interpreted as physical degrees of freedom.

This special role is illustrated already at the level of standard perturbation theory if a bound state is involved. In the calculation of deep inelastic scattering amplitudes, it is the axial or planar gauge (see [11] for the refs.) in which only the planar Feynman graphs contribute to the leading logarithmic asymptotics which allows for a selfconsistent introduction of the hadronic structure functions. In such a gauge, there is a direct correspondence between properly chosen independent fields and the "physical" degrees of freedom. This conceptual necessity of a particular gauge stands in contrast to the usual perturbative analysis where the expansion is restricted to a given order in the coupling constant which makes results explicitly gauge invariant. In that case, the choice of gauge is essentially a technical matter.

Another example of a dynamical scheme where gluons become 'constituents' is the approaches based on the light-cone QCD-Hamiltonian [9]. As in any quantum field theory, a multichannel Hamiltonian must be introduced in order to reproduce, via iterations, the amplitudes which initially are expanded in time ordered Feynman graphs. In the absence of ghosts, wave functions of Fock states with extra dynamical gluons will describe only the "least" number of physical polarizations. In this approach [9], the unitary light cone gauge (for the standard perturbation theory) insures this property.

To apply effectively the multichannel Hamiltonian approach to the dynamics of gluonic ex-

citations in a background we will therefore look for gauges where
i) ghosts are either absent or do not propagate,
ii) there is an unambiguous and simple way to separate in $D_{\mu\nu}(B)$ the non-propagating polarization from the propagating ones.

A gauge satisfying condition i) will be called unitary; it reproduces correctly the number of independent physical propagating polarizations interacting with the background fields. The second requirement gives an additional selection of unitary gauges leading, as we will see, to the most economical use of the Hamiltonian approach.

Gauge invariance obviously allows to eliminate one of the four components of the a_μ-gluons in a background. But since they are generally not 'on mass-shell', it is by no means evident how to formulate a generalized transversality condition by which only two propagating polarizations are retained and which is equivalent to the non-propagation of ghosts. We will show in this work that there are two most suitable unitary gauges and derive the corresponding propagators $D_{\mu\nu}(B)$.

The first one generalizes the ordinary Coulomb gauge propagator of QED and is based on the condition

$$N_i(B)a_i \equiv (D_i(B) + 2D_0^{-1}(B)\hat{F}_{0i}(B))a_i = 0. \tag{2.1}$$

leading to a propagator $D_{\mu\nu}(B)$ which satisfies

$$D_{0i}(B) = 0, \qquad i = 1, 2, 3. \tag{2.2}$$

Here $D_\mu^{ab}(B)$ and $\hat{F}_{\mu\nu}^{ab} \equiv f^{abc}F_{\mu\nu}^c(B)$ are the standard covariant derivative and field strength tensor respectively.

The $D_{00}(B)$ part corresponds to the instantaneous Coulomb exchange modified by the interaction with the background. As for the spatial components $D_{ik}(B)$, we will show that the generalized projector

$$P_{ik}^{ab} = \delta_{ik}\delta^{ab} - (N_i(B)N_l^{-2}(B)N_k(B))^{ab} \tag{2.3}$$

allows one to represent $D_{ik}(B)$ in the sandwiched form

$$D_{ik}(B) = P_{im}(B)(K(B)^{-1})_{mn}P_{nk}(B). \tag{2.4}$$

This gives the correct transversality condition for the propagating physical components orthogonal to the instantaneous polarization in accordance with eq. (2.2).

All other unitary modifications of the Coulomb gauge in the presence of a background do not admit the decoupling condition (2.2) important for the further applications of $D_{\mu\nu}$. As a result, requirement ii) is not satisfied.

The second unitary gauge can be directly obtained from the light cone gauge constraint

$$(n_\mu a_\mu)^a = 0, \qquad n_\mu^2 = 0. \tag{2.5}$$

Other axial gauges (with $n_\mu^2 \neq 0$) being unitary in the presence of background fields do not satisfy condition ii).

The transversality condition of eq. (2.4) for the nonperturbatively interacting gluons has two immediate consequences. First it gives selection rules for the allowed quantum numbers of states with a fixed number of gluonic constituents. A well known example is the (Landau–Yang) theorem for the forbidden total momentum of the system of two real photons. Similarly, this condition decreases the dimensions of multiplets with fixed allowed quantum numbers.

3 Path integral derivation of QED propagator in Coulomb and axial gauges

Let us first review the procedure of gauge fixing performed directly in the path integral representation for the generating functional $Z(J)$.

We will formulate the criterion for a gauge to be ghost free in a way which can be easily generalized to the perturbation theory in a nonabelian background. Our starting point is the standard expression in Euclidean space

$$Z(J) = \int DA_\mu|_{G.F.} \exp\left[-\int d^4x \left(\frac{1}{4g^2}F_{\mu\nu}^2 + J_\mu A_\mu\right)\right] \tag{3.1}$$

and we impose the gauge invariance condition

$$\partial_\mu J_\mu = 0. \tag{3.2}$$

To evaluate $Z(J)$ in the form

$$Z(J) \sim \exp\left[-\frac{g^2}{2}\int d^4x d^4y J_\mu(x) D_{\mu\nu}(x-y) J_\nu(y)\right] \tag{3.3}$$

one must separate explicitly in eq.(3.1) the gauge zero modes $A_\mu^{(0)}$

$$A_\mu^{(0)} = \partial_\mu g \tag{3.4}$$

of the quadratic term $F_{\mu\nu}^2$ or, in other words, fix a gauge. The standard way is to insert into eq. (3.1) the Faddeev–Popov unity in the form

$$1 = det\frac{\delta f(A_\mu^\omega)}{\delta\omega}\int Dg\delta(f(A_\mu^g)) \tag{3.5}$$

where $A_\mu^g = A_\mu + \partial_\mu g$ and $f(A_\mu)$ is the gauge fixing function satisfying $f(A_\mu^\omega) = 0$. Thus

$$Z(J) = \int DA_\mu det\left(\frac{\delta f(A_\mu^\omega)}{\delta\omega}\right)\delta(f(A_\mu))\exp\left[-\int d^4x\left(\frac{1}{4g^2}F_{\mu\nu}^2 + J_\mu A_\mu\right)\right], \tag{3.6}$$

where we omitted the volume of gauge modes $\int Dg$.

By fixing the gauge in this way, the number of independent polarizations in $D_{\mu\nu}$ is always decreased from four to three. There remains the freedom to select one non-propagating polarization.

Rewriting $F_{\mu\nu}^2$ via the kinetic matrix $K_{\mu\nu}$ as

$$2A_\mu K_{\mu\nu} A_\nu$$

and taking for simplicity zero sources $J_\mu = 0$, one gets

$$Z(0) = \left[\frac{det\frac{\delta f}{\delta\omega}}{det K_{ik}}\right]^{1/2}. \tag{3.7}$$

Here, i, k refer to the three components of $K_{\mu\nu}$ selected by $\delta(f(A))$.

Instead of counting independent propagating polarizations of $D_{\mu\nu}$, the absence of ghosts can conveniently be formulated as follows: $detK_{ik}$ should have a factorized form

$$\begin{aligned} detK_{ik} &= det\left(\frac{\delta f}{\delta\omega}\right)\widetilde{det}\left(P_{im}\tilde{K}_{mn}P_{nk}\right) \\ \frac{\partial}{\partial(\partial_\mu^2)}\frac{\delta f}{\delta\omega} &= 0 \end{aligned} \tag{3.8}$$

where the operator $(\frac{\delta f}{\delta\omega})^{-1}$ must be non-propagating and the corresponding determinant in eq.(3.7) must cancel exactly the ghost factor in the numerator of eq. (3.7). We use the notation $\widetilde{det}$ in order to stress that the last determinant in eq. (3.8) denotes formally the result of the integration over the two propagating physical polarizations (selected by the projector P_{ik} ($P_{im}P_{mk} = P_{ik}$)). They determine completely (if eqs.(3.8) are satisfied) the statistical sum $Z(0)$

$$Z(0) = \left[\widetilde{det}\left(P_{im}\tilde{K}_{mn}P_{nk}\right)\right]^{-1/2}. \tag{3.9}$$

In QED, the ghosts decouple always and the first condition of eq. (3.8) holds; only the second one is nontrivial. In the presence of a nonabelian background B_μ, the ghosts interact with the B_μ-fields and factorization (3.8) of the determinant in the statistical sum $Z(0, B)$ is not insured for every gauge. This is related to the fact that in gaussian approximation $Z^{(2)}(0, B)$ of the statistical sum is not invariant under the choice of background gauges for quantum fluctuations. But a background gauge is nevertheless unitary (ghosts do not propagate) if the second condition of eq. (3.8) is met.

In the remainder of this section our aim is to gain experience in how to use the freedom in gauge fixing for bringing the propagator $D_{\mu\nu}(x-y)$ into the form the most convenient for the Hamiltonian technique.

A) Coulomb gauge

Instead of imposing from the begining the gauge condition (3.5) in the form

$$f(A_i) = \partial_i A_i \tag{3.10}$$

one can start from the 'physical' conditions and require that $D_{\mu\nu}$ satisfies

$$\frac{\partial}{\partial(\partial_0^2)}D_{00}^{col}(\partial_\mu) = 0, \quad D_{0i}^{col}(\partial_\mu) = D_{i0}^{col}(\partial_\mu) = 0, \quad i = 1, 2, 3 \tag{3.11}$$

which we would like to preserve in the presence of a background. The second condition of eqs.(3.11) implies that the spacial and temporal components, A_i and A_0, are decoupled, while the first one insures that A_0 does not propagate in time (and is responsible for the instantaneous interaction).

Since there are only two transverse polarizations for ordinary photons, one can impose an additional constraint on $D_{ik}(\partial_\mu)$ via insertion of the gauge unity (3.5) in a form which leads to eqs.(3.11). To understand which function $f(A_\mu)$ brings $D_{\mu\nu}$ into the required form, we first write $\int d^4x \left(\frac{1}{4}F_{\mu\nu}^2\right)$ as

$$\int d^4x \frac{1}{4}F_{\mu\nu}^2 = \frac{1}{2}\int d^4x(-A_0\partial_i^2 A_0 + 2A_0\partial_0\partial_i A_i - \{A_i\partial_\mu^2\delta_{ik}A_k - A_i\partial_i\partial_k A_k\}). \tag{3.12}$$

Comparing with (3.11), we see that the mixed term $A_0\partial_0\partial_i A_i$ should be canceled. This immediately leads to the standard form (3.10) of $f(A_\mu)$.

Summarizing, we see that starting from conditions (3.11) one can selfconsistently determine the form of gauge fixing function $f(A)$ leading to the ghost free gauge. In Section 4 we will generalize this idea to the case of perturbation theory in a nonabelian background.

The main steps leading to $Z(J)$ which will be repeated in the nonabelian case are as follows. In order to get rid of $\delta(\partial_i A_i)$, we introduce the projector on transverse states

$$P_{ik} = \left(\delta_{ik} - \frac{\partial_i\partial_k}{\partial_l^2}\right), \quad \partial_i P_{ik} = 0. \tag{3.13}$$

Decomposing into transverse and longitudinal parts

$$A_i = P_{ik}A_k + (1-P)_{ik}A_k \equiv A_i^\perp + A_i^\parallel, \tag{3.14}$$

one obtains

$$\delta(\partial_i A_i) = (det(-\partial_i^2))^{-1/2}\delta((1-P)_{ik}A_k). \tag{3.15}$$

Therefore, the integration over $(1-P)_{ik}A_k \equiv A_i^\parallel$ is eliminated and $Z(J)$ takes the form

$$\begin{aligned} Z(J) = \int DA_0 DA_i^\perp (det(-\partial_i^2))^{1/2} \exp\left[+\frac{1}{2}\int d^4x \left(\frac{1}{g^2}(A_0\partial_i^2 A_0 + \right.\right. \\ \left.\left. + A_i^\perp \partial_\mu^2 A_i^\perp) - 2(J_0A_0 + J_i^\perp A_i^\perp)\right)\right]. \end{aligned} \tag{3.16}$$

Because only the transverse components $J_i^\perp$ enter, the resulting propagator will have the sandwiched form

$$D_{ik} = P_{il}\tilde{D}_{lm}P_{mk}, \tag{3.17}$$

given after integration over A_0 and $A_i^\perp$ by the final representation (3.3) for $Z(J)$

$$Z(J) = (det(-\partial_\mu^2))^{-1} \exp\left[-\frac{g^2}{2}\int d^4x \left(J_i P_{il}\frac{1}{\partial_\mu^2}P_{lk}J_k + J_0\frac{1}{\partial_i^2}J_0\right)\right]. \tag{3.18}$$

We stress that the remnant of the ghost determinant in eq.(3.16) has been cancelled by the integration over the instantaneous component A_0. The expression for the statistical sum (3.9)

$$Z(0) = (det(-\partial_\mu^2))^{-1} \tag{3.19}$$

is to be interpreted as that for two (transverse) propagating polarizations.

B) Axial gauge with $n^2 \neq 0$

In the abelian case (or in the case of standard nonabelian perturbation expansion) there is another conventional ghost free gauge in which the propagator includes only two propagating polarizations, the well known axial gauge $(nA) = 0$. The gauge fixing function can be conveniently choosen as

$$f(A) = (n\partial)(nA). \tag{3.20}$$

so that the second condition in eq. (3.8), written in terms of light cone variables, holds. Here, n is an arbitrary 4-vector. If it is light-like, the corresponding gauge is the light cone gauge.

All $n^2 \neq 0$ are conceptually equivalent to the elementary cases $n = (1, \vec{0})$ and $n = (0, 1, \vec{0}_\perp)$. The derivation of the propagator is similar in both; we discuss only

$$n_\mu = (1, \vec{0}). \tag{3.21}$$

In full analogy with the previous subsection, one obtains

$$Z(J) = \int DA_i (\det(-\partial_0^2))^{1/2} \exp\left[+\frac{1}{2}\int d^4x (\frac{1}{g^2} A_i K_{ik} A_k - 2J_i A_i)\right] \tag{3.22}$$

where

$$K_{ik} = \partial_\mu^2 \cdot P_{ik} + \partial_0^2 (1-P)_{ik}. \tag{3.23}$$

and the projector P_{ik} is defined by eq. (3.13). The representation (3.23) for K_{ik} provides the separation of nonpropagating and propagating polarizations such that integration over the former cancels the ghost factor $det^{1/2}(-\partial_0^2)$. This separation is analogous to that of eq. (3.16); $A_i^{||}$ represents the nonpropagating component.

Rewriting DA_i as $DA_i^\perp DA^\|$, we arrive at the final representation

$$Z(J) = (det(-\partial_\mu^2))^{-1} \exp\left[-\frac{g^2}{2}\int d^4x \left(J_i (P\frac{1}{\partial_\mu^2}P)_{ik} J_k + J_i((1-P)\frac{1}{\partial_0^2}(1-P))_{ik} J_k\right)\right]. \tag{3.24}$$

In accordance with gauge invariance, $Z(0)$ has the same form (and interpretation) as in the Coulomb gauge. We note that in $D_{\mu\nu}$ the nonpropagating part represented by the last term in the exponent of eq.(3.24) replaces the instantaneous Coulomb exchange part of eq. (3.18).

C) Light cone gauge, $n^2 = 0$

To complete the analysis, we consider also the light cone axial gauge $(nA) = 0$

$$n_- = 1, \quad n_+ = n_\perp = 0 \tag{3.25}$$

where the light cone coordinates for a 4-vector b_μ are $b_\pm = \frac{1}{\sqrt{2}}(b_z \pm b_0)$ and we choose the gauge fixing function f as before.

This choice of n requires a Minkowski metric and straightforward calculations lead to the following analogue of eqs.(3.16) and (3.22)

$$Z(J) = \int DA_- DA_\perp (det(-\partial_+^2))^{1/2} \exp\left[+\frac{i}{2}\int d^4x \left(\frac{1}{g^2} A_\mu K_{\mu\nu} A_\nu + 2J_\mu A_\nu\right)\right] \tag{3.26}$$

where μ, ν run over $+, -, \perp = \{i, k\}$ and

$$K_{++} = -\partial_+^2,\ K_{+i} = K_{i+} = -\partial_+ \partial_i,\ K_{ik} = (2\partial_+\partial_- + \partial_\perp^2)\delta_{ik} - \partial_i \partial_k. \tag{3.27}$$

The form (3.27) of K_{++} implies that the integration over the non-propagating component A_- exactly cancels the ghost factor $(det(-\partial_+^2))^{1/2}$ with the result

$$Z(J) = \int DA_\perp \exp\left[+\frac{i}{2g^2}\int d^4x \left([A_i K_{ik} A_k + 2g^2 J_i A_i] + \right.\right.$$
$$\left.\left. + \{g^2(Jn) - (A_\perp \partial_\perp)(n\partial)\}\frac{1}{(n\partial)^2}\{g^2(Jn) - (n\partial)(\partial_\perp A_\perp)\}\right)\right]. \tag{3.28}$$

Consequently, the final form of $Z(J)$ reads

$$Z(J) = (det(-\partial_\mu^2))^{-1} \exp\left[+\frac{ig^2}{2}\int d^4x\left[(Jn)\frac{1}{(n\partial)^2}(Jn)\ - \right.\right.$$
$$\left.\left. - \left(J_i - (Jn)\partial_i\frac{1}{(n\partial)}\right)\frac{1}{\partial_\mu^2}\left(J_i - \frac{1}{(\partial n)}\partial_i(Jn)\right)\right]\right] \tag{3.29}$$

which corresponds to

$$D_{\mu\nu} = \left[-g_{\mu\nu} + \frac{\partial_\mu n_\nu + \partial_\nu n_\mu}{(n\partial)}\right]\frac{1}{\partial_\mu^2 + i\varepsilon}. \tag{3.30}$$

The sum in the exponent of eq.(3.29) naturally provides us with the separation of propagator (3.30) into the instantaneous

$$D_{\mu\nu}^{inst} = +\frac{n_\mu n_\nu}{(n\partial)^2} \tag{3.31}$$

and propagating parts

$$D_{\mu\nu}^{prop} = \left[-g_{\mu\nu}^\perp + \frac{n_\mu\partial_\nu^\perp + n_\nu\partial_\mu^\perp}{(n\partial)} - \frac{n_\mu n_\nu \partial_\perp^2}{(n\partial)^2}\right]\frac{1}{\partial_\mu^2 + i\varepsilon}. \tag{3.32}$$

This will be useful when we extend our considerations to the nonabelian background. It is easy to check that the transversality condition is satisfied

$$N_\mu D_{\mu\nu}^{prop} = 0, \quad N_\mu = \partial_\mu^\perp + m_\mu(n\partial) \tag{3.33}$$

with the new 4-vector m_μ

$$m_+ = 1, \quad m_- = m_\perp = 0 \tag{3.34}$$

which implies that there are only two propagating polarizations as before.

Summarizing, we see that eliminating in $Z(J)$ one polarization by inserting the gauge unity (3.5), one generates in all three cases a corresponding ghost determinant which is exactly canceled by the integration over the retained physical 'nonpropagating' polarization. In other words, the determinant

$$det(K_{\mu\nu}) \tag{3.35}$$

of the kinetic matrix for the three retained polarizations assumes a factorized form (3.8) where the part which cancels the ghost determinant can be separated from the one describing two propagating physical polarizations.

4 Gauge fixing procedure for the perturbation expansion in a nonabelian background

As in the abelian case, we begin with the path integral form [7, 12] of the generating functional in the presence of a background B_μ

$$Z(J,B) = \int Da_\mu \det\frac{\delta f(B^\omega, a^\omega)}{\delta\omega}|_{\omega_0}\delta(f(B,a))\exp\left[-\int d^4x\left(\frac{1}{4g^2}(F_{\mu\nu}^a(B+a))^2 + J_\mu^a a_\mu^a\right)\right], \tag{4.1}$$

where $f(B^\omega, a^\omega)|_{\omega_0} = 0$ keeping in mind unitary gauges. In what follows the standard notations [12] are used:

$$F_{\mu\nu}(B+a) \equiv F^a_{\mu\nu}T^a = F_{\mu\nu}(B) + D_\mu(B)a_\nu - D_\nu(B)a_\mu - i[a_\mu, a_\nu] \tag{4.2}$$

with

$$D^{ab}_\mu = \partial_\mu\delta^{ab} + f^{abc}B^c_\mu = \partial_\mu\delta^{ab} - i(T^c)^{ab}B^c_\mu \equiv \partial_\mu\delta^{ab} - i(B_\mu)^{ab} \tag{4.3}$$

$$[a_\mu, a_\nu] \equiv a^a_\mu a^b_\nu [T^a, T^b] = i f^{abc} T^c a^a_\mu a^b_\nu. \tag{4.4}$$

For zero sources $J_\mu = 0$, the action density $\sim F^2_{\mu\nu}(B+a)$ obeys the gauge symmetry

$$(B_\mu + a_\mu) \to U(B_\mu + a_\mu + i\partial_\mu)U^{-1} \tag{4.5}$$

which in the infinitesimal form reads

$$B_\mu + a_\mu \to B_\mu + a_\mu + D_\mu(B+a)\omega, \tag{4.6}$$

and where $D_\mu(A)\omega$ can be expressed according to eq.(4.3) as

$$T^a(\partial_\mu\omega^a + f^{abc}\omega^b A^c) \equiv T^a(\partial_\mu\omega^a + (\omega \times A)^a). \tag{4.7}$$

There are two related issues which distinguish the gauge fixing procedure in eq.(4.1) from the one for the standard perturbation theory with no background.

The first one is the well known fact [7] that in the presence of B_μ it is possible to select for the a_μ-fields a gauge which preserves the invariance of $Z(J, B)$ under gauge transformations of B_μ and the sources J_μ

$$B_\mu \to U(B_\mu + i\partial_\mu)U^{-1}, \quad J_\mu \to U J_\mu U^{-1}. \tag{4.8}$$

Usually this invariance is exploited within the background field method [7, 12] to simplify the renormalization scheme and multiloop calculations. Here we would like to stress the integration over background configurations resulting [6] in particular properties of the asymptotics for averaged Wilson loops. To build up the formalism in terms of gauge invariant objects like Wilson loops (with possible spin-insertions, see for example [13]), it is necessary to represent physical amplitudes in terms of gauge invariant combinations of Greens functions in the background fields such as $<\bar\Psi(x)\Psi(y)>$ and $<a_\mu(x)a_\mu(y)>$ for the the matter and the a_μ-fields, respectively. Consequently, it is economical to work with such a gauge for a_μ in which the Greens function $D_{\mu\nu}(x, y|B) =< a_\mu(x)a_\nu(y) >$ is transformed homogeneously with respect to background gauge transformation (4.8)

$$D_{\mu\nu}(x, y|B) \to U(x)D_{\mu\nu}(x, y|B)U^{-1}(y). \tag{4.9}$$

This property follows if $Z(J, B)$ is invariant under (4.8).

The second aspect is the freedom to split the gauge variations of $B_\mu + a_\mu$ field in eight ways between B_μ and a_μ fields:

$$\delta^{(1)}a_\mu = D_\mu(B+a)\omega, \quad \delta^{(1)}B_\mu = 0 \tag{4.10}$$

$$\delta^{(2)}a_\mu = D_\mu(B)\omega, \quad \delta^{(2)}B_\mu = (\omega \times a_\mu) \tag{4.11}$$

$$\delta^{(3)} a_\mu = (\omega \times a_\mu), \quad \delta^{(3)} B_\mu = D_\mu(B)\omega \tag{4.12}$$

$$\delta^{(4)} a_\mu = D_\mu(a)\omega, \quad \delta^{(4)} B_\mu = (\omega \times B_\mu) \tag{4.13}$$

$$\delta^{(5)} a_\mu = (\omega \times B_\mu), \quad \delta^{(5)} B_\mu = D_\mu(a)\omega \tag{4.14}$$

$$\delta^{(6)} a_\mu = \partial_\mu \omega, \quad \delta^{(6)} B_\mu = (\omega \times (B_\mu + a_\mu)) \tag{4.15}$$

$$\delta^{(7)} a_\mu = (\omega \times (B_\mu + a_\mu)), \quad \delta^{(7)} B_\mu = \partial_\mu \omega \tag{4.16}$$

where we omit the eighth irrelevant splitting $\delta^{(8)} a_\mu = 0$.

Since no sufficiently comprehensive discussion of these points exists in the literature [7, 12], we will present the detailed analysis required to construct the nonstandard unitary gauges in the next two sections.

First, we single out the splittings leading to a $Z(J, B)$ which is invariant under (4.8). Since the action of eq.(4.1) itself is invariant under all seven forms of the variation (the third one, (4.12), corresponding to that of eq. (4.8)), possible non invariances must come from the gauge fixing determinant det $\frac{\delta^{(i)} f(B^\omega, a^\omega)}{\delta\omega}$ of the Faddeev-Popov unity in eq. (4.1). Thus, in order to insure invariance under (4.8), $\frac{\delta^{(i)} f}{\delta\omega}$ must transform homogeneously under the infinitesimal variation (4.12) (with infinitesimal parameter $\tilde{\omega}$), i.e.

$$\delta^{(3)} \left(\frac{\delta^{(i)} f(B^\omega, a^\omega)}{\delta\omega} \right) = \left(\tilde{\omega} \times \frac{\delta^{(i)} f(B^\omega, a^\omega)}{\delta\omega} \right). \tag{4.17}$$

For simplicity, we first take covariant gauges and consider the function $f(B, a)$ (linear in a_μ) written in terms tranforming homogeneously under (4.12) such as

$$D_\mu(B) a_\nu, \quad F_{\mu\nu}(B) a_\rho, \ldots . \tag{4.18}$$

This implies that in all interesting cases B_μ enters only inside covariant derivatives $D_\mu(B)$ or field strength tensors $F_{\mu\nu}(B)$.

Consequently, the partial derivatives $\frac{\delta f(B,a)}{\delta B}$ and $\frac{\delta f(B,a)}{\delta a}$ also consist of structures which are transformed homogeneously. As a result, if gauge variations

$$\frac{\delta^{(i)} f}{\delta\omega} = \frac{\delta f}{\delta B}\frac{\delta^{(i)} B}{\delta\omega} + \frac{\delta f}{\delta a}\frac{\delta^{(i)} a}{\delta\omega} \tag{4.19}$$

are to satisfy eq. (4.17), the quantities $\frac{\delta^{(i)} B}{\delta\omega}$ and $\frac{\delta^{(i)} a}{\delta\omega}$ must be also constructed from combinations transforming themselves in accordance with this equation [2].

With this in mind, we condclude that only the three first variations (4.10)–(4.12) lead to an invariant generating functional $Z(J, B)$. In all other cases, $\frac{\delta^{(i)} a}{\delta\omega}$ and $\frac{\delta^{(i)} B}{\delta\omega}$ consist of 'wrong' (in the above sense) combinations of B_μ-fields.

[2] It is not difficult to insure in all interesting cases that indeed there is no "fine tuning" between the two terms of eq.(4.19) and each should satisfy the condition separately.

To illustrate this point, we work out the standard background gauge case

$$f^a(B,a) = (D_\mu(B)a_\mu)^a = 0 \tag{4.20}$$

$$\frac{\delta f^a}{\delta B^b} = f^{acb}a^c = i(T^c a^c)^{ab}, \qquad \frac{\delta f^a}{\delta a^b} = D_\mu(B)^{ab}. \tag{4.21}$$

Combining with eqs.(4.10)

$$\frac{\delta^{(1)} a^a_\mu}{\delta\omega^b} = D_\mu(B+a)^{ab}, \quad \delta^{(1)} B_\mu = 0 \tag{4.22}$$

one gets the standard result [1, 2]

$$\left(\frac{\delta^{(1)} f}{\delta\omega}\right)^{ab} = (D_\mu(B)D^\mu(B+a))^{ab}. \tag{4.23}$$

In a similar way, eqs.(4.11) leading to

$$\frac{\delta^{(2)} a^a_\mu}{\delta\omega^b} = D_\mu(B)^{ab}, \quad \frac{\delta^{(2)} B^a_\mu}{\delta\omega^b} = -i(T^c a^c)^{ab} \tag{4.24}$$

yield

$$\left(\frac{\delta^{(2)} f}{\delta\omega}\right)^{ab} = (D_\mu(B)D_\mu(B) + a_\mu a_\mu)^{ab}, \tag{4.25}$$

while eqs.(4.12)

$$\frac{\delta^{(3)} a^a_\mu}{\delta\omega^b} = -i(T^c a^c)^{ab}, \quad \frac{\delta^{(3)} B^a_\mu}{\delta\omega^b} = D_\mu(B)^{ab} \tag{4.26}$$

implies the final expression

$$\left(\frac{\delta^{(3)} f}{\delta\omega}\right)^{ab} = -i[D_\mu(B), a_\mu]^{ab}. \tag{4.27}$$

Simple but tedious calculations confirm our general conclusion that these three forms of gauge fixing of the a_μ- field do exhaust all possibilites to maintain the invariance of $Z(J,B)$.

We observe that the determinants of the operators (4.23) and (4.25) in the leading order ($a_\mu = 0$) are reduced to

$$det(-D^2_\mu(B)), \tag{4.28}$$

which generalizes the abelian factor (3.19) and represents two propagating ghost polarizations interacting with the background. Note that the determinant of the function (4.27) vanishes formally for $a_\mu = 0$. In the following, we will always use the first kind of gauge splitting (4.10).

In Landau gauge, eq. (4.20), one obtains in leading order

$$Z^{(2)}(0,B) = [\frac{det(-D^2_\mu(B))}{\widetilde{det}(-\tilde{P}_{\mu\rho}(B)(\delta_{\rho\sigma}D^2_\lambda(B) - 2\hat{F}_{\rho\sigma}(B))\tilde{P}_{\sigma\nu}(B))}]^{1/2} \tag{4.29}$$

where $\tilde{P}_{\mu\nu} = \delta_{\mu\nu} - D_\mu \frac{1}{D^2_\lambda} D_\nu$ and the determinant in the denominator indicates the formal integration over three components of a_μ selected by condition (4.20) (see the next section for a similar detailed derivation). We immediately conclude that the conditions of eq. (3.8) are not

satisfied. Ghosts are propagating and in $D_{\mu\nu}$ all three retained polarizations are propagating. Obviously, this gauge is not unitary.

From the above analysis it is clear that the same arguments can be applied to noncovariant gauges to maintain the invariance under (4.8). We must only insure that gauge fixing function $f(B,a)$ is constructed from elements with B_μ entering via the (noncovariant) combinations of long derivatives and field strength tensors. This is exactly what we will exploit when constructing the nonabelian background generalizations of Coulomb and axial gauges.

5 Coulomb background gauge

We first apply the above considerations to obtain the most suitable unitary generalization of the Coulomb gauge for the perturbation theory in a background field which satisfies eq. (3.11). We will also derive the corresponding propagator $D^{col}_{\mu\nu}(B)$ which is required for applications.

As in the abelian case, we start with the quadratic approximation for $F^2_{\mu\nu}(B+a)$ to write the generating functional $Z^{(2)}$ in the form

$$Z^{(2)}(J,B) = \int Da_\mu \det \frac{\delta^{(1)} f(B^\omega, a^\omega)}{\delta\omega}|_{\omega_0} \delta(f(B,a)) \exp\left[+\frac{1}{2}\int d^4x \left(\frac{1}{g^2} a^a_\mu K^{ab}_{\mu\nu}(B) a^b_\nu - 2J^a_\mu a^a_\mu\right)\right] \tag{5.1}$$

where

$$K^{ab}_{\mu\nu} = [D^2_\rho(B)\delta_{\mu\nu} - D_\mu(B)D_\nu(B) - 2\hat{F}_{\mu\nu}(B)]^{ab} \tag{5.2}$$

and

$$\hat{F}^{ab}_{\mu\nu} \equiv f^{abc} F^c_{\mu\nu}. \tag{5.3}$$

We neglected in the expansion of $F^2_{\mu\nu}(B+a)$ the contribution of the term linear in a_μ

$$4a^c_\nu D^{ca}_\mu(B) F^a_{\mu\nu}(B) \tag{5.4}$$

assuming that it merely renormalizes the parameters of the effective action $F^2_{\mu\nu}(B)$ expressed through the proper collective coordinates. This is realized, for instance, in the $1+1$ gas of kinks and antikinks [14]. We stress that this assumption is necessary for the existence of a selfconsistent separation (1.1) of the A_μ field into the two parts.

In order to obtain the gauge fixing function $f(B,a)$ leading to conditions (3.11) we must cancel the mixed terms in $a_\mu K_{\mu\nu} a_\nu$,

$$\begin{aligned} a_0 K_{0i} a_i &= -a_0 D_0 (D_i + 2D_0^{-1}\hat{F}_{0i}) a_i \\ a_i K_{i0} a_0 &= -a_i (D_i + 2\hat{F}_{i0} D_0^{-1}) D_0 a_0 \end{aligned} \tag{5.5}$$

which leads to the following generalization of the abelian condition (3.10)

$$f^c(B,a) = (D_i(B) + 2D_0^{-1}(B)\hat{F}_{0i}(B))^{cd} a^d_i \equiv (N_i(B)a_i)^c. \tag{5.6}$$

The operator $N^{ab}_i(B)$ selects the two transverse 'physical' polarizations. This gauge fixing function yields the second requirement (3.8).

We note that the gauge fixing function $f(B,a)$ of eq.(5.6) can not be obtained by a simple substitution of ∂_i by $D_i(B)$ in the abelian form (3.10).

With this $f(B,a)$, it is straighforward to insert the corresponding gauge unity into the generating functional; we obtain

$$Z^{(2)}(J,B) = \int Da_\mu det(-N_i(B)D_i(B+a))\delta(N_i(B)a_i) \times$$
$$\times \exp\left[+\frac{1}{2}\int d^4x \left\{\frac{1}{g^2}\left(a_0 D_i^2(B)a_0 + a_i K_{ik}(B)a_k\right) - 2J_\mu a_\mu\right\}\right], \tag{5.7}$$

where

$$K_{ik}^{ab}(B) = (\delta_{ik}D_\mu^2(B) - 2\hat{F}_{ik}(B) - D_i(B)D_k(B))^{ab}. \tag{5.8}$$

In order to eliminate $\delta(N_i a_i)$ we introduce, as in the abelian case, the projector

$$P_{ik}(B) = \left(\delta_{ik} - N_i(B)\frac{1}{N_l^2(B)}N_k(B)\right), \;\; N_i P_{ik} = P_{ki}N_i = 0, \tag{5.9}$$

singling out two physical propogating polarizations among the three a_i components. With

$$a_i = P_{ik}a_k + (1-P)_{ik}a_k \equiv a_i^\perp + a_i^\parallel \tag{5.10}$$

and

$$\delta(N_i a_i) = (det(-N_i^2))^{-1/2}\delta(a_i^\parallel), \tag{5.11}$$

one arrives at

$$Z^{(2)}(J,B) = \int Da_0 Da_i^\perp \frac{det(-N_i(B)D_i(B+a))}{det^{1/2}(-N_i^2(B))}\exp\left[\frac{1}{2}\int d^4x \left(\frac{1}{g^2}\{a_0 D_i^2 a_0 \; +\right.\right.$$
$$\left.\left. + \; a_i^\perp(K_{ik} + N_i N_k)a_k^\perp\} - 2J_0 a_0 - 2J_i^\perp a_i^\perp\right)\right], \tag{5.12}$$

where we substituted $K_{ik} \rightarrow \tilde{K}_{ik} = K_{ik} + N_i N_k$ for a later convenience (note that $N_i a_i^\perp = 0$).

Also here, only the transverse current $J_i^\perp = P_{ik}J_k$ enters and thus D_{ik} will have the sandwiched form

$$D_{ik}(B) = P_{im}(B)(\tilde{K}(B)^{-1})_{mn}P_{nk}(B), \tag{5.13}$$

generalizing the abelian expression (3.17). Consequently after integration over $a_0, a_i^\perp$ one obtains the final representation for $Z^{(2)}(J,B)$

$$Z^{(2)}(J,B) = \left\{\frac{det(-N_i(B)D_i(B+a))}{[det(-N_i^2(B))det(-D_i^2(B))]^{1/2}}\right\}(\widetilde{det}(-P_{im}\tilde{K}_{mn}P_{nk}))^{-1/2} \times$$
$$\times \exp\left[-\frac{g^2}{2}\int (J_i D_{ik}J_k + J_0 D_{00}J_0)d^4x\right] \tag{5.14}$$

with

$$D_{00}(x,y|B) = < x|\frac{1}{D_i^2(B)}|y>, \quad D_{0i} = D_{i0} = 0 \tag{5.15}$$

and

$$D_{ik}(x,y|B) = < x|P_{ij}(B)(\delta_{jl}D_\mu^2 - 2\hat{F}_{jl} - D_j D_l + N_j N_l)^{-1}P_{lk}(B)|y> . \tag{5.16}$$

The two equations (5.15),(5.16) are the central result of our paper.

Let us now give a short interpretation of these expressions. We conclude from eq.(5.14) that ghosts are present but do not propagate in time (the factor in curly brakets of eq.(5.14) is equal to unity when $B_\mu = 0$). Indeed the ghost kinetic term $(N_i D_i)$ differs from the corresponding one (D_i^2) for the nonpropagating a_0–component only by the spin dependent part (the last term in eq.(5.6)). In contrast to eqs.(4.28),(4.29) ghost are not propagating and enter only the loop corrections to the instantaneous Coulomb exchange given by D_{00} at the tree level. Consequently, this is a unitary gauge and ghosts do not introduce independent propagating degrees of freedom (described by D_{ik}) which is sufficient for the desired economical formulation of the Hamiltonian approach.

We point out that the expression (5.16) for D_{ik} gives the correct form for both spin-independent and spin-dependent interactions of transverse gluons (decoupled due to conditions (5.15) from the instantaneous component) with the background.

If the conditions (3.11) are not imposed, there is a variety of unitary gauges generalizing the abelian Coulomb one (3.10). Indeed every gauge where $\tilde{N}_i(B)$ has the same limit

$$\tilde{N}_i(B) \to \partial_i \quad \text{for} \quad B_\mu \to 0 \tag{5.17}$$

like $N_i(B)$ and obeys the second condition of eq. (3.8) will lead to nonpropagating ghosts. Because of the coupling between the a_0 and the two retained a_i-components $(D_{0i} \neq 0)$ the choice of the nonpropagating component is not ambigous in this case. It is the simple form (5.15) and (5.16) of $D_{\mu\nu}$ which renders the choice (5.6) of $N_i(B)$ the preferable one for further applications with the multichannel Hamiltonian.

6 Axial background gauges

Next, we derive $Z(J, B)$ in quadratic approximation when the gauge condition is imposed in the form

$$(n_\mu a_\mu)^a = 0. \tag{6.18}$$

As before, the term linear in a_μ (see eq. (5.4)) is neglected also here.

First we analyse whether ghosts are also absent in these gauges if there is a background. For this purpose it is sufficient to consider $Z^{(2)}(0, B)$ and look for a representation in the form of eq.(3.9). It will be shown that only the light cone $(n^2 = 0)$ axial gauge remains ghost free in the presence of an arbitrary background. Again we will obtain the propagator and give a short interpretation.

A. The case $n^2 \neq 0$

As in the abelian case, for $n^2 \neq 0$ it is sufficient to consider $n = (1, \vec{0})$ or $n = (0, 1, \vec{0}_\perp)$. Both elementary choices are similar, we take only the temporal gauge

$$n = (1, \vec{0}), \tag{6.19}$$

for which in Euclidean space

$$Z^{(2)}(0, B) = \int Da_i det^{1/2}(-D_0^2(B)) \exp[+\frac{1}{2} \int d^4x a_i K_{ik}(B) a_k], \tag{6.20}$$

with

$$K^{ab}_{ik}(B) = [(D_0^2(B) + D_l^2(B))\delta_{ik} - D_i(B)D_k(B) - 2\hat{F}_{ik}(B)]^{ab}. \tag{6.21}$$

We note that in this gauge the ghost factor $det^{1/2}(-D_0^2(B))$ does not depend on the a_μ-fields. Equation (6.20) immediately leads to

$$Z^{(2)}(0,B) = \left[\frac{det(-D_0^2(B))}{det(-K_{ik}(B))}\right]^{1/2}. \tag{6.22}$$

Again we see that $Z^{(2)}(0,B)$ is different for different background gauges because the gaussian approximation to $Z(0,B)$ implicit in perturbation theory is not invariant under a change of the background gauges. This raises the question whether there is a 'best' gauge.

As in the abelian case, the absence of ghosts is equivalent to the existence of a projector

$$P^{ab}_{ik}(B) = \delta^{ab}\delta_{ik} - \left(M_i(B)\frac{1}{M_l^2(B)}M_k(B)\right)^{ab}, \quad P^2 = P$$

$$a_i = M_i\frac{1}{M_l^2}(M_k a_k) + P_{ik}a_k \tag{6.23}$$

such that the following generalization of the first condition in (3.8) holds (the second one is obviously satisfied)

$$det(-K_{ik}) = det(-D_0^2)\widetilde{det}(-P_{im}K'_{mn}P_{nk}). \tag{6.24}$$

In other words, ghosts are absent if there exists an operator $M_i^{ab}(B)$ such that the result of integration over the instantaneous component defined as $b = \frac{1}{\sqrt{M_i^2}}\,(M_k a_k)$ $(Da_i = DbD(Pa)_\perp)$ cancels exactly the ghost factor. The last condition results in

$$M_i D_0^2 \delta_{ik} M_k = M_i K_{ik} M_k \tag{6.25}$$

which means in turn that the 3-dimentional kinetic matrix $\tilde{K}_{ik} = K_{ik} - \delta_{ik}D_0^2$

$$\tilde{K}_{ik} = D_l^2\delta_{ik} - D_iD_k - 2\hat{F}_{ik} \tag{6.26}$$

has a continuum of zero modes $\left(M_i\frac{1}{\sqrt{M_l^2}}b\right)$. This finally leads to the constraint

$$det[-\tilde{K}_{ik}] = 0 \tag{6.27}$$

reproducing the condition (6.25).

To demonstrate that eq.(6.25) can not be satisfied without an extra constraint on $F_{\mu\nu}(B)$ let us consider first the simpler $2+1$ case where

$$det[-\tilde{K}_{ik}] \sim det(-D_1^2 + (D_1D_2 + \hat{F}_{21})D_2^{-2}(D_2D_1 + \hat{F}_{12})). \tag{6.28}$$

The operator in eq.(6.28) can be represented as

$$(\hat{F}_{12}D_2^{-1}D_1 - D_1D_2^{-1}\hat{F}_{12}) + \hat{F}_{12}D_2^{-2}\hat{F}_{12}, \tag{6.29}$$

and one concludes that it vanishes in general only if

$$[D_i, \hat{F}_{ik}] = 0, \quad i,k = 1,2. \tag{6.30}$$

This is not surprising because $\tilde{K}_{ik}$ of eqs.(6.26), (6.28) coincides with the kinetic operator (5.2) for the 1+1 case and therefore has gauge zero modes $D_i\omega$ [1, 2] in the presence of a background which satisfies the classical $1+1$ equations of motion (6.30). In the $2+1$ case at hand, any classical background satisfies

$$[D_i, \hat{F}_{ik}] = -[D_0, \hat{F}_{0k}] \tag{6.31}$$

which is equivalent to eq. (6.30) only if in addition

$$[D_0, \hat{F}_{0k}] = 0 \tag{6.32}$$

is imposed.

It is not difficult to show that in 3+1 dimensions, eqs. (6.30) with $i, k = 1, 2, 3$ are necessary to satisfy eq.(6.25). Therefore, for $n^2 \neq 0$ it is impossible to cancel the ghost factor exactly for an arbitrary background field, even if it is only classical. Thus we conclude that in general the background axial gauge (6.18) is not ghost-free when $n^2 \neq 0$.

Still, for all gauges where $M_i^2(B)$ satisfies the second condition (3.8) and

$$\frac{1}{\sqrt{M_k^2(B)}} M_i(B) \to \frac{1}{\sqrt{\partial_k^2}} \partial_i, \quad B_\mu \to 0 \tag{6.33}$$

one gets

$$Z^{(2)}(0, B) = \left\{ \frac{det(-D_0^2) det(-M_i^2)}{det(M_i K_{ik} M_k)} \right\}^{1/2} \widetilde{det}^{-1/2} [(PK'P)_{ik}] \tag{6.34}$$

and the factor in curly brakets becomes unity for $B_\mu = 0$ when we recover the abelian form (3.19). Consequently, the only propagating part of $Z^{(2)}(0, B)$ arises from the integration over $(P_{ik}a_k)$ and ghosts lead merely to loop corrections to the nonpropagating contribution of the exchange. As a result, all such gauges (6.33) can be called unitary.

The impossibility to cancel $det^{1/2}(-D_0^2)$ corresponds to the ambiguity in the definition of the projector P_{ik} onto transverse propagating polarizations since all $M_i(B)$ of eq.(6.33) are suitable (if the second condition (3.8) is not violated). Consequently, one can not select unambigously the non-propagating component from the rest.

B. Light cone gauge $n^2 = 0$

In this gauge with

$$n_- = 1, \quad n_+ = n_i^\perp = 0, \tag{6.35}$$

the quadratic approximation to $Z(J, B)$ is given by

$$\begin{aligned} Z^{(2)}(J, B) = & \int Da_- Da_i \{det(-D_+^2(B))\}^{1/2} \times \\ & \times exp\left[+\frac{i}{2} \int d^4x \left\{ \frac{1}{g^2} a_\mu K_{\mu\nu}(B) a_\nu + 2(J_+ a_- + J_i a_i) \right\} \right], \end{aligned} \tag{6.36}$$

where

$$\begin{aligned} & K_{++} = -D_+^2, \ K_{+i} = -D_+ D_i - 2\hat{F}_{+i}, \ K_{i+} = -D_i D_+ - 2\hat{F}_{i+}, \\ & K_{ik} = D_\mu^2 \delta_{ik} - D_i D_k - 2\hat{F}_{ik} \end{aligned} \tag{6.37}$$

and i, k stand for the two perpendicular components.

From these equations one obtains after integration over the a_- component

$$Z^{(2)}(J,B) = \int Da_i \exp\left[+\frac{i}{2g^2}\int d^4x\{a_i\tilde{K}_{ik}a_k - 2g^2a_i(K_{i+}K^{-1}_{++}J_+ - J_i)\}\right]\times$$
$$\times \exp\left[-\frac{ig^2}{2}\int d^4x J_+(K_{++})^{-1}J_+\right], \qquad (6.38)$$

where

$$\tilde{K}^{ab}_{ik} = K^{ab}_{ik} - (K_{i+}(K_{++})^{-1}K_{+k})^{ab}. \qquad (6.39)$$

We see that the ghost factor in eq. (6.36) is cancelled exactly by the integration over a_- as in the abelian case. The absence of ghosts in the light cone background gauge makes it the most convenient one (among the axial gauges) for the Hamiltonian approach built up in terms of physical polarizations alone.

Integrating out the a_i-fields, one obtains $Z^{(2)}(J,B)$ in the representation of eq.(3.3)

$$Z^{(2)}(J,B) = \{\det\tilde{K}_{ik})\}^{-1}\exp\left\{-\frac{ig^2}{2}\int d^4x(L_i(J)\tilde{K}^{-1}_{ik}L_k(J) + J_+(K_{++})^{-1}J_+)\right\} \qquad (6.40)$$

where

$$L_i(J) = J_i - K_{i+}(K_{++})^{-1}J_+ \qquad (6.41)$$

The physical interpretation of eq.(6.40) is similar to the abelian case (3.29). The determinant in the preexponent reproduces the statistical sum $Z(0,B)$ in term of two propagating polarizations. In the exponent itself, the first term gives the part of the propagator $D^{prop}_{\mu\nu}(B)$ responsible for these two propagating polarizations. The second term supplies us with the modified instantaneous exchange

$$D^{inst}_{\mu\nu}(B) = \frac{n_\mu n_\nu}{D^2_+(B)} \qquad (6.42)$$

generalizing the abelian expression (3.31).We note that the choice of physical propagating polarizations is unambiguous.

Similarly to $D_{ik}(B)$ of eq.(5.16) in the Coulomb background gauge, the piece $D^{prop}_{\mu\nu}(B)$ given by

$$D^{prop}_{\mu\nu}(B) = -(g^\perp_{\mu\rho} - n_\mu(nKn)^{-1}(nK)^\perp_\rho)\tilde{K}^{-1}_{\rho\sigma}(g^\perp_{\sigma\nu} - (Kn)^\perp_\sigma(nKn)^{-1}n_\nu) \qquad (6.43)$$

provides, in particular, the light cone representation for the spin-dependent interactions of physical polarizations with the background. We note that it is more complicated than in the Coulomb case.

7 Applications of unitary gauges

As pointed out in the introduction, the basic motivation of our work is to design a dynamical scheme to investigate QCD bound states including gluonic excitations at the scale of confinement ("constituent gluons"). In this section we sketch how the propagators can be used to this aim.

In order to simplify the discussion and to separate the confining dynamics we are interested in here from the effects of chiral symmetry breaking for light quarks, we will consider the case of spinless quarks only. For heavy quarks the spin-dependent interactions can be related [13] via

a cluster expansion to the same irreducible correlators of background fields which constitute the averaged Wilson loop. For light quarks these interactions are connected to chiral symmetry breaking and an additional information on the confining configurations is required. Work in this direction is now in progress.

The Feynman-Schwinger representation allows one to write the Greens function of the $q\bar{q}$ state in a path integral form. For spinless quarks in the quenched approximation we have [6]

$$G = \int ds_1 Dz_1 ds_2 Dz_2 \exp[-K_1 - K_2] < W(C) >_{B+a} \tag{7.1}$$

where $< W(C) >_{B+a}$ is the Wilson loop operator averaged over all gluonic fields $A_\mu = B_\mu + a_\mu$

$$\begin{aligned} < W(C) >_{B+a} = \frac{1}{Z} \int DBDa \det \frac{\delta^{(1)} f(B^\omega, a^\omega)}{\delta\omega}|_{\omega_0} \delta(f(B,a)) \times \\ \times \exp\left[-\frac{1}{4g_0^2}\int F^2_{\mu\nu}(B+a) d^4x\right] P\exp\left[i\int_C (B_\mu + a_\mu) dx_\mu\right] \end{aligned} \tag{7.2}$$

and the integration over DB_μ implies a summation over all relevant collective coordinates excluded from Da_μ in the standard way (we omit for simplicity the corresponding orthogonality conditions ensured via the Faddeev–Popov trick). The contour C consists of quark and antiquark trajectories z_1, z_2 and K_1 and K_2 are the standard quark kinetic terms [6] whose explict form will not be required here.

Quantum fluctuations a_μ can be taken into account by expanding the path ordered exponent $P\exp[i\int_C(B_\mu + a_\mu)dx_\mu]$ in powers of a_μ. The result can be represented as

$$< W(C) >_{B+a} = < W(C) >_B + \sum_{n=1}^{\infty} i^n \int \left\langle W^{(n)}(C; x(1), \ldots, x(n)) \right\rangle_B dx(1) \ldots dx(n) \tag{7.3}$$

where $< W^{(n)} >_B$ is a Wilson loop with n insertions of the a_μ fields [4]. For example, the quantity

$$\begin{aligned} &\int < W^{(2)} >_B dx(1)dx(2) = \\ &= g^2 \int dx_\mu(1) dx_\nu(2) \left\langle \Phi^{\alpha\beta}_{C_1}(x(1), x(2)|B) D^{\delta\alpha;\beta\gamma}_{\mu\nu}(x(1), x(2)|B) \Phi^{\gamma\delta}_{C_2}(x(2), x(1)|B) \right\rangle_B + \\ &+ O(g^3) \end{aligned} \tag{7.4}$$

(with $D^{\delta\alpha;\beta\gamma}_{\mu\nu} = t^{\delta\alpha}_a t^{\beta\gamma}_b D^{ab}_{\mu\nu}$) describes (apart from the $O(g^3)$ corrections) the one gluon exchange in Fig. 1a. Here Φ_{C_i} denotes necessary parallel transporters in the fundamental representation along the subpathes C_i of the initial contour C

$$\Phi_{C_i}(x, y|B) = P\exp\left[i\int_x^y (B^a_\mu t^a) dx_\mu\right] \tag{7.5}$$

while $D_{\mu\nu}$ is the propagator of the fluctuations in the background.

We emphasize that the subscript B in eqs. (7.3), (7.4) refers to retaining only the B_μ-field in all relevant path ordered exponents. The averaging procedure is still performed with the action density $F^2_{\mu\nu}(B+a)$ as in eq. (7.2).

To incorporate $< W^{(n)}(C) >_B$ consistently into the bound state formalism, a multichannel Hamiltonian approach has been proposed [10]. Within this approach, time ordered diagrams as

in eq.(7.3) can be reproduced by iterations of the effective Hamiltonian. The problem is thus reformulated as that of a relativistic many body system with mixed channels.

Under general circumstances, a selfconsistent application of the multichannel Hamiltonian formalism to a quantum field theory requires [9] the light cone frame (coordinates). For this purpose, the covariant generalization [15] of the Coulomb gauge (3.10) can be used where the new propagator takes the form (5.15), (5.16) in the co-moving Minkowski coordinates, obtained by a Lorentz transformation in accordance with the total velocity of the system V_μ

$$n^{(0)}_\mu = V_\mu = \gamma(1, V, 0_\perp), \quad n^{(1)}_\mu = \gamma(V, 1, 0_\perp), \quad n^{(3,4)}_\mu = (0, 0, \vec{n}_\perp). \tag{7.6}$$

One can express $n^{(\alpha)}_\mu$ via the conventional light cone coordinates to recover the conceptually important suppression of pair creation from the vacuum. In the limit $V \to 1$, the new D_{00} part expressed in terms of light cone coordinates corresponds to the instantaneous Coulomb exchange boosted from the rest frame to the infinite momentum one [15]. Consequently, it makes sense to work with $D^{Col}_{\mu\nu}(B)$ in the form of eqs.(5.15), (5.16), which is, in addition, somewhat simpler than $D_{\mu\nu}(B)$ of eqs.(6.42),(6.43) in the light cone gauge. Keeping this in mind, we are led to the following strategy [10].

Take bound states with light quarks where the long distance dynamics clearly dominates. Then one can treat in Euclidean space the hard and soft parts of the a_μ-field separately as fast and slow subsystems. First one integrates the hard part with four-momentum squared $p^2_\mu \geq \frac{1}{T^2_g}$ where T_g is the scale at which the area law of $< W(C) >_B$ starts and the effective string between gluons and quarks forms.

Apart from the inducing new effective vertices, this also leads to a renormalization of the running coupling constant at the scale $\frac{1}{T_g}$ where it freezes due to the confining vacuum [4]. With this effective action the soft part (with Euclidean four-momentum squared $p^2_\mu \leq \frac{1}{T^2_g}$) of the a_μ-field is handled, together with the B_μ-field, in the following way.

At the initial step one must take into account the contributions from the first term in eq.(7.3), $< W(C) >_B$, plus the sum over all orders in the instantaneous exchange $D_{00}(x, y|B) \sim \delta(x_+ - y_+)$ from the rest. In the following, we will always replace the integration over the background fields B_μ by the sum over irreducible correlators of the field strength $F_{\mu\nu}(B)$ with the help of the cluster expansion [6]. At the hadronic scale, this enables one to use the expansion in $\frac{T_g}{<r>}$ where $T_g \sim 0.2 - 0.4\ fm$ [6] is the decay length of the correlators and $< r >$ is the caracteristic distance between constituents. In this way one retains [6] the first few local terms (such as the area and perimeter law contributions) for quantities like the Wilson loop averaged over B_μ–fields only

$$< W(C) >_B = exp(-\sigma S + \rho l + ...). \tag{7.7}$$

where S, l, σ and ρ are the area, length, string tension and effective mass respectively. We note that at the level of the vacuum average, the contribution of the higher derivative structures (such as the curvature) is relatively suppressed for large smooth countours.

Denoting this part of $< W(C) >_{B+a}$ as $< W^{(q\bar{q})}(C) >_{B+a}$, one can introduce the effective $q\bar{q}$ Lagrangian defined formally by

$$\int_0^T L(q\bar{q})dt_+ = -\ln\left[\int ds_1 Dz_1 ds_2 Dz_2 \exp[-K_1 - K_2] < W^{(q\bar{q})}(C) >_{B+a}\right] \tag{7.8}$$

After continuing to Minkowski space, the Legendre transformation gives us the Hamiltonian $H(q\bar{q})$ for the two valence quark Fock sector $\Psi(q\bar{q})$.

We emphasize that $L(q\bar{q})$ contains on the light cone both the frozen flux tube contribution [17] arising from the area law asymptotics (7.7) of $< W(C) >_B$ and also the soft part of the instantaneous Coulomb exchange determined by $D_{00}(B)$ which describes the interactions with the vacuum fields. Diagramatically, it corresponds to Fig.2.

At the next stage, one can calculate those time ordered diagrams of $< W(C) >_{B+a}$ which are obtained by the iterations of the the two-coupled channel Hamiltonian

$$H^{(2)} = \left(\begin{array}{ll} H(q\bar{q}) & V(q\bar{q} \to q\bar{q}g) \\ V(q\bar{q}g \to q\bar{q}) & H(q\bar{q}g) \end{array} \right) \tag{7.9}$$

acting on a Fock state vector

$$\Psi = \left(\begin{array}{l} \Psi(q\bar{q}) \\ \Psi(q\bar{q}g) \end{array} \right) \tag{7.10}$$

which includes also the state with one valence gluon (see Fig.1b). For the case $B_\mu = 0$ this approximation to H has been considered in ref. [16].

Generally, $H^{(2)}$ must be evaluated according to the scheme which we discuss at the example of the simplest subset of the D_{ik} iterations in Fig. 3. Time intervals with the propagating gluons are described by the Hamiltonian $H(q\bar{q}g)$ of the first excitation of the string 'frozen' during the rest of the time. It can be determined already from the simple diagram of Fig. 4a (where the valence gluon propagates during the entire time) in the same way [17] as $H(q\bar{q})$. This diagonal ansatz, for an arbitrary number of gluons has been suggested recently in [5]. We note that in order to calculate $H(q\bar{q}g)$ fully, one must take into account all instantaneous exchanges between valence quarks and the gluon (as in Fig. 4b).

Without these corrections, the elements of $H(q\bar{q}g)$ are given by the Legendre transformation of the Lagrangian matrix $L_{ik}(q\bar{q}g)$ (continued to Minkowski space) which is formally given by

$$\int_0^T L_{ik}(q\bar{q}g)dt_+ = -\ln\left[ds_1 Dz_1 ds_2 Dz_2 \exp[-K_1 - K_2] < \Phi(x,y|B) D_{ik}(x,y|B)\Phi(y,x|B) >_B\right] \tag{7.11}$$

corresponding to Fig. 4a. Apart from the kinetic and the spin-dependent terms, there are at large distances two frozen strings (Fig. 1b) connecting the a_μ-gluon with the quark and antiquark, respectively. They appear explicitely after using the Feynman-Schwinger representation of $D_{ik}(B)$ which implies a path ordered exponent in the adjoint representation along the gluon trajectories over which the path integral is performed [4]. Cluster expansion techniques [18] allow to reformulate the averaging in eq. (7.11) as an average for two Wilson loops in the fundamental representation with the closed contours $(C_1 + C_g)$, $(C_1 - C_g)$ bounded by the corresponding quark and gluon trajectories. As a result, the area law asymptotics for these Wilson loops $< W(C_1 \pm C_g) >_B$ induces at large distances a (frozen) string between the gluon and each quark. The sandwiched form (eq. (5.13)) insures that there are only two transverse propagating polarizations. Note that eq. (7.11) relates D_{ik} of eq. (5.16) to both spin-independent and spin-dependent effective interactions of the valence gluon.

Non-diagonal elements $V(q\bar{q}g \to q\bar{q})$ can be obtained uniquely from the requirement that the amplitude of Fig. 1a is reproduced in the second iteration of $H^{(2)}$ (see ref. [10] for details).

For the case $B_\mu = 0$, the Hamiltonian $H^{(2)}$ would lead [16] to the existence of a continuum part of the spectrum. Here the inclusion of the confining background allows to invoke (on light cone in particular) the confining QCD string [17] and its excitations which is still not elaborated in the standard Hamiltonian approach [9] (see [19] for a discussion).

In principle, one can continue this procedure to include step by step higher order Fock states which are relevant for a problem at hands. There are some indications [10] that there exists, at least for the low lying bound states a dynamical parameter (in addition to the moderate value of coupling constant α_{st} frozen [4] due to the confining background) which justifies this expansion.

Acknowledgments

This paper was prepared mainly during a few visits of one of the authors (A.D.) to the University of Zurich. He would like to thank all the staff of the Institute of Theoretical Physics for warm hospitality extended to him. This work is supported by Schweizerischer Nationalfonds, by the Russian Fundamental Research Foundation (A.D.), grant N 93–02–14937, and by INTASS.

References

[1] G.'tHooft, Phys. Rev. **D14** (1976) 3432.

[2] A.Polyakov, Nucl. Phys. **B120** (1977) 429,
"Gauge fields and strings" Harwood academic publishers, 1987.

[3] G.'tHooft, Nucl. Phys. **B190** (1981) 455.

[4] Yu.Simonov, Preprint HD–THEP–93–16.

[5] Yu.Simonov in Proceedings of Hadron'93, p.2629, edited by T.Bressani et. al.

[6] H.Dosch, Yu.Simonov, Phys. Lett. **B205** (1988) 339,
Yu.Simonov, Yad. Fiz. **54** (1991) 192.

[7] B.S.DeWitt, Phys. Rev. **162** (1967) 1195, 1239;
G.'tHooft, Nucl. Phys. **B62** (1973) 444;
G.'tHooft in Acta Universitatis Wratislavensis no.38, 12th Winter School of Theoretical Physics in Karpacz; Functional and probabilistic methods in quantum field theory. v.I (1975).

[8] I.Tamm, J. Phys. (U.S.S.R.) **9** (1945) 449;
S.Dancoff, Phys. Rev. **78** (1950) 382.

[9] S.Brodsky and H.Pauli, in Recent Aspects of Quantum Fields, H.Mitter et. al., Eds., Lecture Notes in Physics, v.396 (Springer–Verlag, Berlin, 1991);
K.Wilson et. al., Phys. Rev. **D49** (1994) 6720.

[10] A.Dubin, in preparation.

[11] Yu.Dokshitzer, D.Dyakonov, S.Troyan, Phys. Rep. **58** (1980) 269.

[12] L.Abbott, Nucl. Phys. **B185** (1981) 189;
W.Dittrich et. al., Phys. Lett. **B128** (1983) 321, Phys. Lett. **B144** (1984) 99.

[13] Yu.Simonov, Nucl. Phys. **B324** (1989) 67.

[14] E.Bogomolny, Phys. Lett. **91B** (1980) 431.

[15] A.Dubin, J.Tjon, in preparation.

[16] M.Krautgärtner, H.Pauli, F.Wölz, Phys. Rev. **D45** (1992) 3755.

[17] A.Dubin, A.Kaidalov, Yu.Simonov, Phys. Lett. **B343** (1995) 310.

[18] A.Dubin, Yu. Kalashnikova, preprint ITEP-40-94, hep-ph 9406332, Yad. Fiz (in press).

[19] R.Perry, Lectures presented at Hadrons'94, Gramado, April, 1994.

Figure captions

Fig.1a One a_μ–gluon exchange contribution to $< W(C) >_{B+a}$.

Fig.1b Modes of the QCD string arising (on light cone) in the amplitude of Fig.1a.

Fig.2 The part of $< W(C) >_{B+a}$ responsible for the dynamics in the valence quark Fock sector.

Fig.3 The simplest subset of exchanges from the part of $< W(C) >_{B+a}$ relevant for the two Fock states approximation (7.9) to the Hamiltonian.

Fig.4a The simplest amplitude for reconstruction of $H(q\bar{q}g)$.

Fig.4b Amplitudes with the instantaneous exchanges leading to a corrected $H(q\bar{q}g)$.

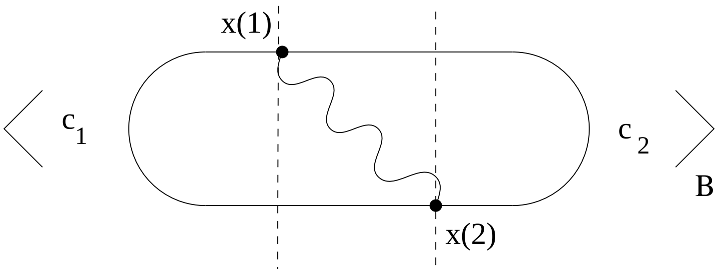

Fig. 1a

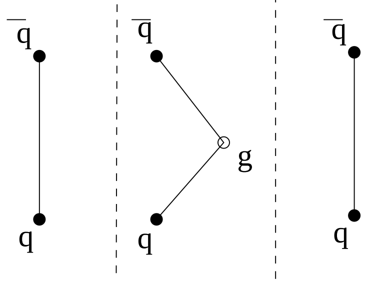

Fig. 1b

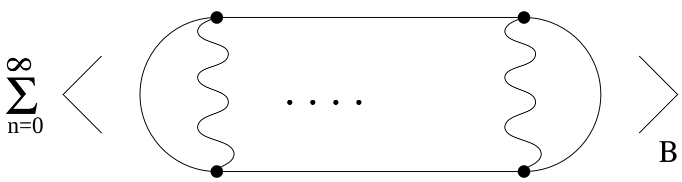

Fig. 2

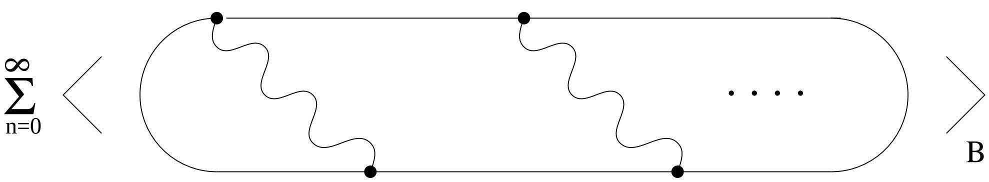

Fig. 3

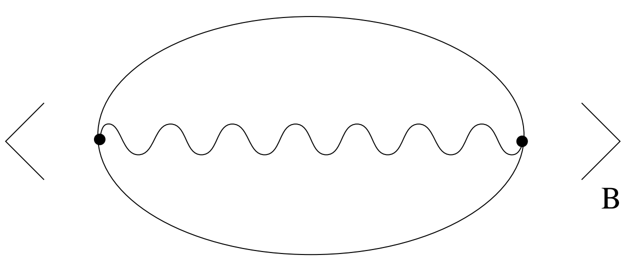

Fig. 4a

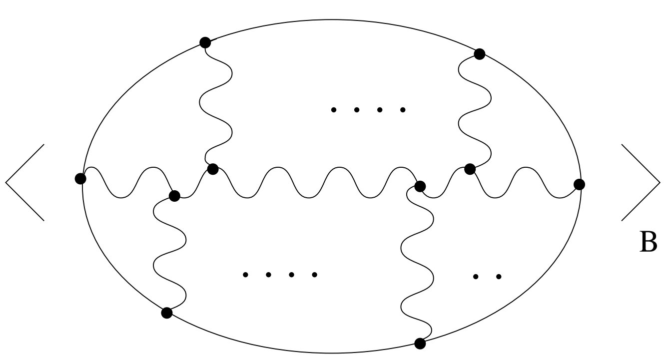

Fig. 4b